

Lepton energy moments in semileptonic charm decays

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We search for signals of Weak Annihilation in inclusive semileptonic D decays. We consider both the widths and the lepton energy moments, which are quite sensitive probes. Our analysis of Cleo data shows no clear evidence of Weak Annihilation, and allows us to put bounds on their relevance in charmless B semileptonic decays.

I. INTRODUCTION

The value of $|V_{ub}|$ preferred by current global analyses of CKM data is about 15% smaller than the one extracted from inclusive charmless semileptonic B decays [1]. Though not very significant, the discrepancy has prompted a reexamination of the sources of theoretical uncertainty in the inclusive determination [2]. Weak Annihilation (WA) contributions are generally considered an important source of uncertainty in the Operator Product Expansion (OPE) that describes the inclusive B decays [3], and affect especially the high q^2 and lepton endpoint analyses. They appear in the OPE as $1/m_b^3$ corrections involving the matrix elements of dimension-6 four-quark operators, and affect both the total $B \rightarrow X_u \ell \bar{\nu}$ decay rate and the charged lepton energy spectrum [4, 5].

Early estimates of the relevant matrix elements were derived in the framework of QCD sum rules [6]. They were also computed on the lattice in the static heavy quark limit [7] and with propagating heavy quarks [8]. However, it was soon realized [9, 10] that the light flavour component of the WA operators can differ from the light valence quark of the B -meson leading to the so-called non-valence WA contributions which are extremely difficult to study non-perturbatively. In addition, contrary to valence WA contributions, the latter cannot be constrained by comparing charged and neutral B meson semileptonic decay rates and a common prejudice that the former should dominate may be unfounded.

It was also noted in ref.[10–12] that the WA matrix elements that enter $B \rightarrow X_u \ell \bar{\nu}$ decay can be constrained via the semileptonic decays of D and D_s mesons, using heavy quark symmetry. Several authors have attempted to extract information on WA contributions from the measured total semileptonic rates of $D^{0,+}$ and D_s , most recently in [13–15]. For instance, one may attribute the observed differences in $D^{+,0}$ and D_s semileptonic widths [16]

$$\Gamma(D^+ \rightarrow X e^+ \nu) / \Gamma(D^0 \rightarrow X e^+ \nu) = 0.985(28) , \quad \Gamma(D_s^+ \rightarrow X e^+ \nu) / \Gamma(D^0 \rightarrow X e^+ \nu) = 0.828(57) \quad (1)$$

to the valence spectator quark WA contributions in D_s decays, since they are Cabibbo suppressed in the D^+ case and completely absent in D^0 decays. However, additional contributions to this difference arise from $SU(3)$ breaking in the matrix elements of all dimension 5 and 6 operators, that contribute significantly to the total rates [11]. Moreover, the slow convergence of the $1/m_c$ expansion, the generally large perturbative corrections to the semileptonic charm width, and the strong dependence on the charm mass may obscure the determination of the non-valence WA component from the widths¹.

Our strategy in this paper is to consider also the moments of the lepton energy spectra in the OPE and compare them with recent experimental results from Cleo [16]. Not only are the moments free of the strong dependence on the charm quark mass and its associated uncertainty, but their perturbative and non-perturbative corrections tend to cancel as well. Moreover, since WA is expected to dominate the spectrum endpoint, leptonic moments might be more sensitive to WA contributions than the total rate. A study of moments in D semileptonic decays is also instrumental for a critical reassessment of the OPE in charm decays many years after [17], in view of the recent experimental results and of the successful application to B semileptonic decays.

In the next Section, we present the experimental results on the leptonic spectra and compute from them the first leptonic moments. The OPE calculation of the moments is presented in Section III, while we derive and discuss our results in Section IV. We conclude with a brief summary.

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¹ For a related discussion on the charm meson lifetimes see [11].

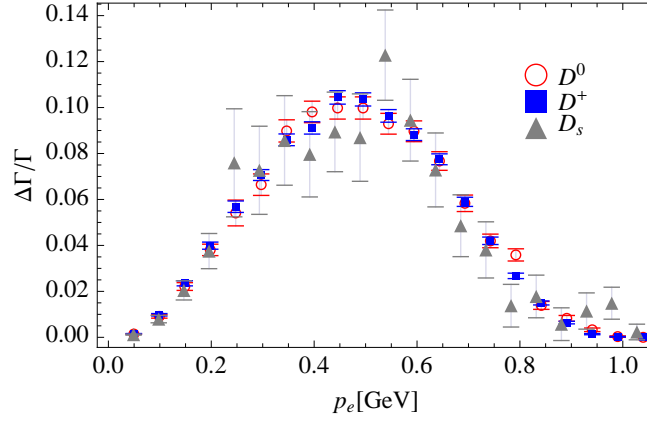


Figure 1: Normalized charm semileptonic decay spectra [16], extrapolated to $p_e = 0$ and boosted to the reference frame of the decaying particle.

II. EXPERIMENTAL DATA

Recently the Cleo Collaboration has measured the electron spectra of inclusive semileptonic $D^{+,0}$ and also D_s decays with a lower cut on the electron momentum in the lab frame of $p_e > 0.2$ GeV [16]. They extract the total decay rates by extrapolating the spectra over the remaining phase-space, using a theoretically modeled sum over known exclusive modes. Unfortunately, they do not provide higher leptonic energy moments.

Cleo give the electron energy spectra in the laboratory frame. Since the decaying $D^{0,+}$ mesons are pair-produced with center-of-mass energy $E_{CM} = 3.774$ GeV, they are boosted in this frame with $\beta = 0.14, 0.15$, respectively. In the case of D_s mesons, the situation is more complicated since roughly half of them are produced in the primary vertex at $E_{CM} = 4.170$ GeV associated with D_s^* mesons ($\beta = 0.2$). The other half then comes from radiative and hadronic D_s^* decays.

We extract the moments without cuts by first extrapolating the measured spectra towards $p_e = 0$. We use a procedure similar to the one employed by Cleo themselves, but instead of using a sum over exclusive modes with model form factors, we use the theoretical form of the spectrum in the $p_e \approx 0$ region, where the OPE is expected to provide a satisfactory description. In particular, for $p_e \rightarrow 0$ the spectrum must die at least like p_e^2 . Thus we fit the first four measured bins to $d\Gamma/dx = ax^2(1+bx)(1-x)$, with $x = 2E_e/m_c$. In the case of D_s , due to the larger statistical uncertainties, we set $b = 0$ and fit only to the first two measured bins. In the fits we take the reported systematic errors to be fully correlated between different bins. Using the fitted formulas we have computed the rates and obtained the following branching fractions

$$\mathcal{B}(D^0 \rightarrow X e \nu) = 0.064(1), \quad \mathcal{B}(D^+ \rightarrow X e \nu) = 0.161(3), \quad \mathcal{B}(D_s \rightarrow X e \nu) = 0.065(4), \quad (2)$$

fully compatible with Cleo reported results. We then compute in the lab frame the first two leptonic energy moments normalized to the total rate, again assuming the systematic errors to be fully correlated among different bins and obtain

$$\langle E_e \rangle_{lab}^{D^0} = 0.465(3) \text{ GeV}, \quad \langle E_e^2 \rangle_{lab}^{D^0} = 0.248(2) \text{ GeV}^2, \quad (3)$$

$$\langle E_e \rangle_{lab}^{D^+} = 0.459(1) \text{ GeV}, \quad \langle E_e^2 \rangle_{lab}^{D^+} = 0.242(1) \text{ GeV}^2, \quad (4)$$

$$\langle E_e \rangle_{lab}^{D_s} = 0.466(12) \text{ GeV}, \quad \langle E_e^2 \rangle_{lab}^{D_s} = 0.254(13) \text{ GeV}^2. \quad (5)$$

The dependence of these values upon our extrapolation ansatz is expected to be smaller than the corresponding effect in the total rate, since the contribution of the $p_e \sim 0$ region to the n -th (unnormalized) moment is $(2p_e/m_D)^n$ suppressed compared to its contribution to the total rate. Thanks to the correlations between numerators and denominators, the total errors in (5) are significantly smaller than in (2).

In order to compare these values to theoretical predictions in the D meson frame one still needs to take into account the boost factors. The energy of the final state electron in the lab frame is given by $E'_e = \gamma E_e (1 - \beta \cos \theta)$, where E_e is the electron energy in the D meson frame and θ is the angle between the momentum of the electron and the one of the D meson in the lab frame. We thus obtain that $\langle E'_e \rangle \equiv \langle E_e \rangle_{lab} = \gamma \langle E_e \rangle$ with $\gamma = 1.009, 1.012$ for Cleo's $D^{+,0}$ mesons, respectively. Similarly, for the 49% of D_s 's which come from the primary vertex, $\gamma \simeq 1.02$. The remaining

51% receive an additional boost in an arbitrary direction since they originate from D_s^* decays $D_s^* \rightarrow D_s \gamma$ (94%) or $D_s^* \rightarrow D_s \pi$ (6%). In view of the precision in (5), the effect of this additional boost is always negligible. The directional averaging for the second electron energy moment $\langle E_e^2 \rangle$ results in $\langle E_e'^2 \rangle = \langle \gamma^2 (1 + \beta^2/3) \rangle \langle E_e^2 \rangle$ which again can be readily computed for all cases. The extrapolated rest-frame spectra are displayed in Fig. 1. Our results for the moments in the D mesons rest frame are

$$\begin{aligned} \langle E_\ell \rangle_{exp}^{D^0} &= 0.459(3) \text{GeV}, & \langle E_\ell^2 \rangle_{exp}^{D^0} &= 0.240(2) \text{GeV}^2, & \langle (E_\ell - \langle E_\ell \rangle)^2 \rangle_{exp}^{D^0} &= 0.029(2) \text{GeV}^2, \\ \langle E_\ell \rangle_{exp}^{D^+} &= 0.455(1) \text{GeV}, & \langle E_\ell^2 \rangle_{exp}^{D^+} &= 0.236(1) \text{GeV}^2, & \langle (E_\ell - \langle E_\ell \rangle)^2 \rangle_{exp}^{D^+} &= 0.029(1) \text{GeV}^2, \\ \langle E_\ell \rangle_{exp}^{D_s} &= 0.456(11) \text{GeV}, & \langle E_\ell^2 \rangle_{exp}^{D_s} &= 0.239(12) \text{GeV}^2, & \langle (E_\ell - \langle E_\ell \rangle)^2 \rangle_{exp}^{D_s} &= 0.031(12) \text{GeV}^2. \end{aligned} \quad (6)$$

Here we have also listed the variances (σ_E^2) of the spectra. Within the stated uncertainties there is no sign of a difference between the moments of D_s and $D^{0,\pm}$. This is at odds with what one would naively expect from eq.(1). However, as mentioned in the Introduction, the concurring contribution of $SU(3)$ violation in the matrix elements of higher dimensional operators might provide a partial explanation.

One can estimate the order of magnitude of $SU(3)$ breaking corrections by comparing the hyperfine splitting of $D^{0,+}$ and D_s mesons, $\Delta_{D_q}^{hf} = 3(m_{D_q^*}^2 - m_{D_q}^2)/4$ (which is related to the OPE parameter μ_G^2):

$$\Delta_{D^+}^{hf} = 0.409(1) \text{GeV}^2, \quad \Delta_{D^0}^{hf} = 0.413(1) \text{GeV}^2, \quad \Delta_{D_s}^{hf} = 0.440(2) \text{GeV}^2. \quad (7)$$

We see that even isospin violation between $D^{+,0}$ mesons is manifest, at the expected 1% level. On the other hand $SU(3)$ violation is sizable and of the order 10%. Lattice QCD studies of f_D and f_{D_s} [18] suggest that even 20% violation can be expected in certain quantities. Since the $1/m_c^2$ and $1/m_c^3$ contributions to the total semileptonic rate can be as large as 50% of the leading order estimate (depending on the charm quark scheme and scale), and since one can expect $SU(3)$ violation of similar size in the matrix elements of the relevant power suppressed operators, an $O(10\%)$ $SU(3)$ breaking in the widths is not unlikely. In order to reliably extract possible WA contributions from the measured total semileptonic rates of D and D_s mesons [16], one would therefore need to estimate the size of $SU(3)$ violation in all these matrix elements. In the case of normalized moments, on the other hand, some of the leading power corrections cancel out and one might be more directly sensitive to WA contributions.

III. LEPTONIC MOMENTS IN THE OPE

The perturbative corrections to moments of some kinematic distributions in semileptonic b decays are now known through $O(\alpha_s^2)$ [20]. In the case of semileptonic charm decays, however, only the $O(\alpha_s)$ [25] and $O(\beta_0 \alpha_s^2)$ corrections are readily available using [21]. For what concerns the power corrections, explicit expressions for the leptonic spectrum at $O(1/m_c^2)$ can be found in the last paper of [3], while we have computed the $O(1/m_c^3)$ contributions from the form factors given in [22]. We neglect $O(1/m_c^4)$ corrections [23] and $O(\alpha_s/m_c^2)$ corrections, for which only the μ_π^2 contribution to the rate is known.

We calculate the semileptonic rate and the total moments from the lepton energy spectra in terms of $x \equiv 2E_e/m_c$, where E_e is the electron energy in the D meson inertial frame. For simplicity, we only consider total leptonic moments, without lower cuts on the lepton energy, and neglect the lepton mass. We define

$$\begin{aligned} \Gamma^{(n)} \equiv \int_0^{(1-r)} \frac{d\Gamma}{dx} x^n dx &= \frac{G_F^2 m_c^5}{192\pi^3} |V_{cs}|^2 \left[f_0^{(n)}(r) + \frac{\alpha_s}{\pi} f_1^{(n)}(r) + \frac{\alpha_s^2}{\pi^2} f_2^{(n)}(r) + \frac{\mu_\pi^2}{m_c^2} f_\pi^{(n)}(r) + \frac{\mu_G^2}{m_c^2} f_G^{(n)}(r) \right. \\ &\quad \left. + \frac{\rho_{LS}^3}{m_c^3} f_{LS}^{(n)}(r) + \frac{\rho_D^3}{m_c^3} f_D^{(n)}(r) + \frac{32\pi^2}{m_c^3} B_{WA}^{(n)s} \right], \end{aligned} \quad (8)$$

where $r = m_s^2/m_c^2$, $\alpha_s \equiv \alpha_s(m_c)$, and $\mu_{\pi,G}^2$, $\rho_{D,LS}^3$ are the D meson matrix elements of the dimension 5 and 6 local operators appearing in the OPE. $B_{WA}^{(n)s}$ is the WA contribution to the n -moment: as we will explain in a moment, it is related to the matrix elements of four-quark operators. In addition, there are also Cabibbo suppressed contributions, which can be included in the analysis by using the above formula in the limit $r \rightarrow 0$ and by replacing V_{cs} with V_{cd} and similarly $B_{WA}^{(n),s}$ with $B_{WA}^{(n),d}$. They contribute to the total rate at the level of 5%, but their effect is highly suppressed in the normalized moments, with the possible exception of WA contributions.

The tree-level spectrum in charm semileptonic decays is softer than in the analogous b decays and peaks at $x \approx 0.6$, a feature qualitatively evident already in Fig. 1. The lowest order expressions for the rate and the first two moments

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$$\begin{aligned} f_0^{(0)} &= 1 - 8r + 8r^3 - r^4 - 12r^2 \log r, \\ f_0^{(1)} &= [2r^5 - 15r^4 + 60r^3 - 20r^2 - 60r^2 \log r - 30r + 3]/5, \\ f_0^{(2)} &= [-r^6 + 8r^5 - 30r^4 + 80r^3 - 35r^2 - 60r^2 \log r - 24r + 2]/5. \end{aligned}$$

The associated $O(\alpha_s)$ corrections are known in closed form only for the total rate. We employ accurate numerical interpolations valid in the range $0.04 < \sqrt{r} < 0.2$ in the pole mass scheme

$$\begin{aligned} f_1^{(0)}(r) &= 2.86\sqrt{r} - 3.84r \log r - 2.47, \\ f_1^{(1)}(r) &= 2.11\sqrt{r} - 2.43r \log r - 1.56, \\ f_1^{(2)}(r) &= 1.66\sqrt{r} - 1.71r \log r - 1.09. \end{aligned}$$

Similarly, while there is an analytic result for $f_2^{(0)}(r)$ as an expansion in r [20], we employ only the BLM approximation [21] in the form of numerical interpolations in the pole mass scheme:

$$\begin{aligned} f_2^{(0)}(r) &= \beta_0[8.16\sqrt{r} - 1.21r \log r - 3.38], \\ f_2^{(1)}(r) &= \beta_0[6.15\sqrt{r} - 0.35r \log r - 2.24], \\ f_2^{(2)}(r) &= \beta_0[4.99\sqrt{r} + 0.11r \log r - 1.64], \end{aligned}$$

where β_0 is the QCD beta function $\beta_0 = 11 - 2n_f/3$ and $n_f = 3$ is the number of light flavors. The above expressions can be translated to any other scheme (and associated IR scale) using the condition that the decay rate be scheme (and scale) independent at each perturbative order together with the known scale dependence of the heavy quark mass and power suppressed non-perturbative parameters. For the leading power corrections we get

$$\begin{aligned} f_\pi^{(0)}(r) &= -f_0^{(0)}(r)/2, & f_\pi^{(1)}(r) &= 0, & f_\pi^{(2)}(r) &= \frac{5}{6}f_0^{(2)}(r), \\ f_G^{(0)}(r) &= \frac{1}{2}f_0^{(0)}(r) - 2(1-r)^4, & f_G^{(1)}(r) &= \frac{1}{3}f_0^{(1)}(r) - \frac{6}{5}(1-r)^5, & f_G^{(2)}(r) &= \frac{1}{6}f_0^{(2)}(r) - \frac{4}{5}(1-r)^6, \\ f_{LS}^{(0)}(r) &= -f_G^{(0)}(r), & f_{LS}^{(1)}(r) &= \frac{8}{5}(1-r)^5, & f_{LS}^{(2)}(r) &= \frac{5}{6}f_0^{(2)}(r) + \frac{4}{3}(1-r)^6, \\ f_D^{(0)}(r) &= \frac{77}{6} + O(r) + 8 \log \frac{\mu_{\text{WA}}^2}{m_c^2}, & f_D^{(1)}(r) &= \frac{78}{5} + O(r) + 8 \log \frac{\mu_{\text{WA}}^2}{m_c^2}, \\ f_D^{(2)}(r) &= \frac{87}{5} + O(r) + 8 \log \frac{\mu_{\text{WA}}^2}{m_c^2}, \end{aligned}$$

where μ_{WA} is the $\overline{\text{MS}}$ renormalization scale associated to the mixing of the Darwin and WA operators, an $O(\alpha_s^0)$ effect [26, 27]. The μ_{WA} dependence cancels against the implicit scale dependence of $B_{\text{WA}}^{(n),q}(\mu_{\text{WA}})$. A change of μ_{WA} therefore shifts part of the Darwin operator contribution into the WA contribution, with important consequences for the error analysis. We find that for values of μ_{WA} below 1GeV the size of the ρ_D^3 coefficient in the width is comparable to that of other power corrections.

WA contributions can be identified with the matrix elements of the dimension-6 four quark operators

$$\mathcal{O}_1^{Qq} = \bar{Q}\gamma_\mu(1-\gamma_5)q\bar{q}\gamma^\mu(1-\gamma_5)Q \quad \text{and} \quad \mathcal{O}_2^{Qq} = \bar{Q}(1-\gamma_5)q\bar{q}(1-\gamma_5)Q,$$

where Q is the heavy quark. The matrix elements that enter the charm meson decay rates are defined as

$$B_{\text{WA}}^q = \frac{1}{2m_D} \langle D | \mathcal{O}_2^q - \mathcal{O}_1^q | D \rangle = \frac{1}{2}m_D f_D^2 (B_{D,2}^q - B_{D,1}^q), \quad (9)$$

where $B_{D,i}^q$ parameterize the deviation from the factorization approximation: B_{WA}^q vanishes in the limit of factorization. As it was recognized long ago [4], WA is localized at the endpoint of the lepton energy spectrum, and can be approximately expressed by a delta function at the partonic endpoint. In this case, we would expect $B_{\text{WA}}^{(n),q} = B_{\text{WA}}^q$, for all n , up to small $O(r)$ effects. In fact, gluon bremsstrahlung and hadronization effects are expected to smear the WA contribution over a region of electron energy around the partonic threshold $(m_c^2 - m_s^2)/2m_c$ corresponding

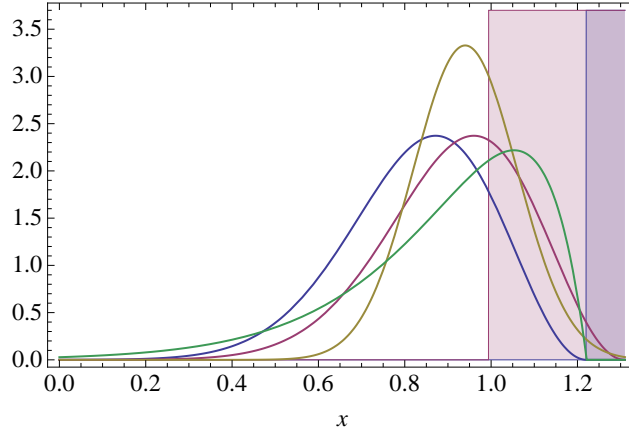


Figure 2: Different distributions of WA as a function of $x = 2E_e/m_c$, for $m_c = 1.4\text{GeV}$. The distributions extend to $x = 1.22$ corresponding to the η threshold, or to $x = 1.31$, corresponding to the 2π threshold for $D^{+,0}$.

to $x = 1 - r$. The size and shape of the smearing may affect the various integrals $B_{\text{WA}}^{(n),q}$ differently and cannot be predicted, although we expect a small perturbative tail to emerge away from the endpoint. Clearly, smearing towards smaller E_e tends to suppress the WA contributions to higher lepton energy moments, in which case we expect $B_{\text{WA}}^{(n+1),q} \lesssim B_{\text{WA}}^{(n),q}$. We also know that the WA contributions involve a $SU(3)$ flavor singlet in the final state, and may be confined below the two-pion or η thresholds which, for D decays and $m_c = 1.4\text{GeV}$, correspond to $x \leq 1.31$ and $x \leq 1.22$, respectively. For D_s decays the allowed region is slightly larger. Indeed, the WA distributions spread over a region of electron energies of $O(\Lambda_{QCD})$. In Fig. 2 we show four possible forms of the WA distribution as a function of x , assuming only positivity and the presence of a tail towards lower x . Two of the forms displayed stop at the η threshold, the other two extend to the two-pion threshold.

The endpoint behavior of WA can also be parameterized in terms of matrix elements of local four-quark operators of higher and higher dimension [19], which determine the WA distribution. At leading order in the heavy quark expansion, however, the parameters $B_{\text{WA}}^{(n),q}$, i.e. the total moments of the WA distribution, are all determined by the dimension six term, and therefore they are all equal to $B_{\text{WA}}^{(0),q}$. The smearing is therefore a power and α_s -suppressed effect in $B_{\text{WA}}^{(n),q}$, but it may be phenomenologically important despite the formal suppression.

We will be primarily interested in a determination of the leading matrix element, $B_{\text{WA}}^{(0),q}$, i.e. the zeroth moment of the WA distribution, from $B_{\text{WA}}^{(n),q}$: the non-negligible smearing of WA may lead to a model-dependent dilution. In order to quantify this effect on the parameters $B_{\text{WA}}^{(1),q}$, we have considered a number of distributions like those in Fig. 2. In general the dilution is a moderate effect: $B_{\text{WA}}^{(1),q} \gtrsim 0.8 B_{\text{WA}}^{(0),q}$, $B_{\text{WA}}^{(2),q} \gtrsim 0.7 B_{\text{WA}}^{(0),q}$, and of course for WA distributions localized to the right of the partonic endpoint, $B_{\text{WA}}^{(n>0),q} \geq B_{\text{WA}}^{(0),q}$. However, distributions characterized by a longer tail will lead to a stronger dilution, which may also be enhanced in the normalized moments. In the next Section, when we try to extract information on the WA from the leptonic moments, we will carefully take this effect into account.

The WA contributions to the decays of charmed mesons involving different spectator quarks can be parametrically decomposed as follows:

$$\begin{aligned} \Gamma_{\text{WA}}^{(n)}(D^0) &\propto \cos^2 \theta_c B_{\text{WA}}^{(n),s}(D^0) + \sin^2 \theta_c B_{\text{WA}}^{(n),d}(D^0), \\ \Gamma_{\text{WA}}^{(n)}(D^+) &\propto \cos^2 \theta_c B_{\text{WA}}^{(n),s}(D^+) + \sin^2 \theta_c B_{\text{WA}}^{(n),d}(D^+), \\ \Gamma_{\text{WA}}^{(n)}(D_s) &\propto \cos^2 \theta_c B_{\text{WA}}^{(n),s}(D_s) + \sin^2 \theta_c B_{\text{WA}}^{(n),d}(D_s), \end{aligned} \quad (10)$$

where θ_c is the Cabibbo angle. In the isospin limit, the number of independent contributions is reduced by the identification $B_{\text{WA}}^{(n),s}(D^0) = B_{\text{WA}}^{(n),s}(D^+)$. Any difference in the moments between the D^0 and the D^+ can only be due to isospin violation or Cabibbo suppressed contributions. Since the data do not show any significant difference in the rates or the moments of D^0 and D^+ in what follows we will neglect such Cabibbo suppressed effects. We are then left with just two distinct contributions, which can be identified with the valence and non-valence WA terms involving the s quark. The first one, $B_{\text{WA}}^{(n),s}(D_s)$, only contributes to the D_s decays, while the second one, $B_{\text{WA}}^{(n),s}(D)$, contributes equally to D^+ and D^0 decays. In the flavor $SU(3)$ limit, the two contributions correspond to the sum of isotriplet and isosinglet, and to the isosinglet contribution, respectively.

IV. RESULTS AND DISCUSSION

Our first task will be to check whether the OPE at $O(1/m_c^3)$ describes the experimental data in a satisfactory way. In the pole mass scheme with $m_c = 1.6\text{GeV}$, $r = 0.005$, and $\mu_{\text{WA}} = 0.8\text{GeV}$ we have

$$\Gamma = \Gamma_0 \left[1 - 0.72\alpha_s - 0.29\alpha_s^2\beta_0 - 0.60\mu_G^2 - 0.20\mu_\pi^2 + 0.42\rho_D^3 + 0.38\rho_{LS}^3 + 80B_{\text{WA}}^{(0)} \right], \quad (11)$$

$$\langle E \rangle = \langle E \rangle_0 \left[1 - 0.03\alpha_s - 0.03\alpha_s^2\beta_0 - 0.07\mu_G^2 + 0.20\mu_\pi^2 + 1.4\rho_D^3 + 0.29\rho_{LS}^3 + 135\bar{B}_{\text{WA}}^{(1)} \right], \quad (12)$$

$$\langle E^2 \rangle = \langle E^2 \rangle_0 \left[1 - 0.07\alpha_s - 0.05\alpha_s^2\beta_0 - 0.14\mu_G^2 + 0.52\mu_\pi^2 + 3.5\rho_D^3 + 0.66\rho_{LS}^3 + 204\bar{B}_{\text{WA}}^{(2)} \right], \quad (13)$$

$$\sigma_E^2 = (\sigma_E^2)_0 \left[1 - 0.09\alpha_s - 0.05\alpha_s^2\beta_0 - 0.14\mu_G^2 + 1.7\mu_\pi^2 + 9.4\rho_D^3 + 1.4\rho_{LS}^3 + 641\bar{B}_{\text{WA}}^{(\sigma)} \right], \quad (14)$$

where $\beta_0 = 9$, Γ_0 , $\langle E \rangle_0$, $\langle E^2 \rangle_0$, $(\sigma_E^2)_0$ are the tree-level results, and all coefficients are in GeV to the appropriate power. We immediately notice that the lepton moments receive smaller perturbative corrections in the pole mass scheme than the total rate, while they are very sensitive to WA and ρ_D^3 . Since the uncertainty on the value of ρ_D^3 to be employed in Eqs. (11-14) is sizable, it follows from the previous section that the choice of μ_{WA} has a strong impact on the final error on WA. Indeed, the simultaneous use of all three observables can reduce this ambiguity. As for the m_c dependence, since $\langle E^n \rangle$ scales like m_c^n , the lowest moments are less sensitive to the value of the charm mass than the width, which is proportional to m_c^5 . We have also introduced effective WA parameters $\bar{B}_{\text{WA}}^{(1)} \simeq B_{\text{WA}}^{(1)} - \frac{3}{5}B_{\text{WA}}^{(0)}$, $\bar{B}_{\text{WA}}^{(2)} \simeq B_{\text{WA}}^{(2)} - \frac{2}{5}B_{\text{WA}}^{(0)}$, $\bar{B}_{\text{WA}}^{(\sigma)} \simeq B_{\text{WA}}^{(0)} - 3.8B_{\text{WA}}^{(1)} + 3.2B_{\text{WA}}^{(2)}$.

In order to avoid the well-known problems associated to the pole mass, semileptonic decays are usually described in terms of *threshold* heavy quark masses, like the kinetic mass, the PS, or the 1S mass, see [2] for a review. In the following we focus on the kinetic scheme [28]. For the charm mass and the non-perturbative OPE parameters we use as initial inputs the results of a global fit to $B \rightarrow X_c \ell \nu$ and radiative moments [29],

$$\begin{aligned} m_c(1\text{GeV}) &= 1.16(5)\text{GeV}, & \mu_\pi^2(1\text{GeV}) &= 0.44(4)\text{GeV}^2, \\ \rho_{LS}^3(1\text{GeV}) &= -0.19(8)\text{GeV}^3, & \rho_D^3(1\text{GeV}) &= 0.19(2)\text{GeV}^3. \end{aligned}$$

The above values refer to expectation values of local operators in the B meson in the kinetic scheme with the IR cutoff scale $\mu_{\text{kin}} = 1\text{GeV}$. Up to power corrections, the D meson expectation values can be identified with the B meson ones, but $\mu_{\text{kin}} = 1\text{GeV}$ is too high compared to the charm mass and we need to run the parameters to a lower IR scale above Λ_{QCD} .

We evolve the OPE parameters including the charm mass down to $\mu_{\text{kin}} = 0.5\text{GeV}$ using $O(\alpha_s^2)$ expressions. The expectation values μ_G^2 and ρ_{LS}^3 do not run with the kinetic scale. The running to such low scales induces significant uncertainties in the other parameters, of both perturbative and non-perturbative origin, which we estimate by varying the scale of α_s . We finally adopt $m_c(0.5\text{GeV}) = 1.40(7)\text{GeV}$, $\mu_\pi^2(0.5\text{GeV}) = 0.26(6)\text{GeV}^2$, $\rho_D^3(0.5\text{GeV}) = 0.05(4)\text{GeV}^3$. Of course, the charm mass determination we use is not the most precise. For instance, Ref. [30] reports a very precise value in the $\overline{\text{MS}}$ scheme which is consistent with [29]. However, any scheme translation would increase the error significantly. In the end, we do not expect an improvement even in the case of the width. In addition, we assume that the large uncertainty on ρ_{LS}^3 as extracted from the B fit dominates over the perturbative and non-perturbative corrections to its value. In the case of μ_G^2 , on the other hand, we use a conservative approach with the central value of [31] and a large error $\mu_G^2 = 0.35(10)\text{GeV}^2$. For the strange quark we use its $\overline{\text{MS}}$ definition, $m_s(2\text{GeV}) = 0.105(2)\text{GeV}$ [32]. We evaluate the corrections to the total rate and the first two leptonic moments at $\mu_{\text{WA}} = 0.8\text{GeV}$ as

$$\Gamma_{\text{kin}} = 1.2(3)10^{-13}\text{GeV} \left\{ 1 + 0.23\alpha_s + 0.18\alpha_s^2\beta_0 - 0.79\mu_G^2 - 0.26\mu_\pi^2 + 1.45\rho_D^3 + 0.56\rho_{LS}^3 + 120B_{\text{WA}}^{(0)} \right\}, \quad (15)$$

$$\langle E_\ell \rangle_{\text{kin}} = 0.415(21)\text{GeV} \left\{ 1 + 0.03\alpha_s + 0.02\alpha_s^2\beta_0 - 0.09\mu_G^2 + 0.26\mu_\pi^2 + 2.7\rho_D^3 + 0.44\rho_{LS}^3 + 203\bar{B}_{\text{WA}}^{(1)} \right\}, \quad (16)$$

$$\langle E_\ell^2 \rangle_{\text{kin}} = 0.192(20)\text{GeV}^2 \left\{ 1 + 0.001\alpha_s + 0.02\alpha_s^2\beta_0 - 0.18\mu_G^2 + 0.68\mu_\pi^2 + 6.6\rho_D^3 + 0.99\rho_{LS}^3 + 307\bar{B}_{\text{WA}}^{(2)} \right\}, \quad (17)$$

$$\sigma_{E,\text{kin}}^2 = 0.019(2)\text{GeV}^2 \left\{ 1 - 0.53\alpha_s - 0.17\alpha_s^2\beta_0 - 0.18\mu_G^2 + 2.2\mu_\pi^2 + 17\rho_D^3 + 2.1\rho_{LS}^3 + 961\bar{B}_{\text{WA}}^{(\sigma)} \right\}, \quad (18)$$

where all coefficients are in GeV to the appropriate power and the tree-level results include the error due to their mass dependence. In the first and the second leptonic energy moment, the dominant source of uncertainty is by far ρ_D^3 , while in the total rate the charm quark mass uncertainty dominates. In view of the accuracy relevant to our analysis, we can turn a blind eye to the bad convergence of the expansion for the width. Varying all the input parameters as

described above and α_s by $\pm 20\%$ around $\alpha_s(m_c) = 0.36$ to account for higher order perturbative effects, we compare the theoretical expressions with eqs. (2,6) and we extract the following values of $B_{\text{WA}}^{(0)}(D_q)$ and $\bar{B}_{\text{WA}}^{(1,2,\sigma)}(D_q)$

$$B_{\text{WA}}^{(0),s}(D^0) = -0.001(3)\text{GeV}^3, \quad B_{\text{WA}}^{(0),s}(D^+) = -0.001(3)\text{GeV}^3, \quad B_{\text{WA}}^{(0),s}(D_s) = -0.002(3)\text{GeV}^3, \quad (19)$$

$$\bar{B}_{\text{WA}}^{(1),s}(D^0) = -0.0001(6)\text{GeV}^3, \quad \bar{B}_{\text{WA}}^{(1),s}(D^+) = -0.0001(6)\text{GeV}^3, \quad \bar{B}_{\text{WA}}^{(1),s}(D_s) = -0.0001(6)\text{GeV}^3, \quad (20)$$

$$\bar{B}_{\text{WA}}^{(2),s}(D^0) = -0.0001(10)\text{GeV}^3, \quad \bar{B}_{\text{WA}}^{(2),s}(D^+) = -0.0002(10)\text{GeV}^3, \quad \bar{B}_{\text{WA}}^{(2),s}(D_s) = -0.0001(10)\text{GeV}^3, \quad (21)$$

$$\bar{B}_{\text{WA}}^{(\sigma),s}(D^0) = -0.0000(7)\text{GeV}^3, \quad \bar{B}_{\text{WA}}^{(\sigma),s}(D^+) = -0.0000(7)\text{GeV}^3, \quad \bar{B}_{\text{WA}}^{(\sigma),s}(D_s) = 0.0001(10)\text{GeV}^3. \quad (22)$$

At the estimated theoretical precision, all the extracted WA contributions are consistent with zero. The obvious implication is that the OPE describes all the data reasonably well. We also repeated the exercise at different kinetic scales, finding in general similar results, despite the high sensitivity to accidental cancellations. For larger μ_{kin} the size of perturbative corrections to the rate and their associated error increase rapidly as noticed in [34]. However, the moments are less affected and one obtains consistent WA estimates for kinetic scales as large as $\mu_{\text{kin}} = 0.8 \text{ GeV}$ with similar errors. We have also repeated our analysis using the 1S scheme for the charm mass [33] and $\mu_{\text{kin}} = 0$ for the other OPE parameters. The apparent convergence of the perturbative expansion improves for the rate and variance, but the estimates and errors of the WA contributions are very similar.

Eqs.(19-22) show no departure from the $SU(3)$ flavor symmetry, but the errors are strongly correlated among different mesons. On the other hand, in the ratios of D_s and D^0 semileptonic widths and leptonic moments, the dominant m_c and all the perturbative corrections cancel, while only isotriplet WA and $SU(3)$ breaking contributions to the OPE parameters remain. We estimate the latter conservatively as 20% of the central values and add the resulting errors linearly to account for possible correlations. Comparing these expressions to the experimental values we obtain $\Delta B_{\text{WA}}^{(0),s} \equiv B_{\text{WA}}^{(0),s}(D_s) - B_{\text{WA}}^{(0),s}(D^0) = -0.0014(12)(5)\text{GeV}^3$ from the ratio of rates, $\Delta \bar{B}_{\text{WA}}^{(1),s} = 0.0000(3)(1)\text{GeV}^3$ from the ratio of the first moments, and $\Delta \bar{B}_{\text{WA}}^{(2),s} = 0.0000(4)(2)\text{GeV}^3$ from the ratio of the second leptonic energy moments, where we show the theoretical and experimental uncertainty contributions in the first and the second brackets respectively. We observe a mild indication of a nonzero negative isotriplet $B_{\text{WA}}^{(0),s}$ contribution. This indication is however not really significant at present.

So far we have parameterized the WA contributions to the individual moments, and we have learned that the OPE gives a satisfactory description of the data even in the absence of WA, with the possible exception of $\Delta B_{\text{WA}}^{(0),s}$. As already mentioned in the previous Section, we expect WA to be concentrated at the end-point, in which case we can combine the $D^{0,+}$ results from eqs. (20,22) to obtain the isosinglet $B_{\text{WA}}^{(0),s} = -0.0003(15) \text{ GeV}^3$. In the worst case, the reduction in sensitivity due to the WA smearing functions shown in Fig. 2 is about 60%, which leads to

$$B_{\text{WA}}^s = -0.0003(25)\text{GeV}^3, \quad (23)$$

where we have combined the dilution error linearly, without changing the central value. We regard this as sufficiently conservative. We can also extract $\Delta B_{\text{WA}}^{(0),s}$ from $\Delta \bar{B}_{\text{WA}}^{(1,2),s}$: in the absence of WA smearing they both give the same result, $\Delta B_{\text{WA}}^{(0),s} = 0.0000(7)(3)$, while the worst dilution scenario gives $\Delta B_{\text{WA}}^{(0),s} = 0.0000(12)(3)$, which is compatible with the value extracted from the ratio of widths.

We can also look for indications of WA dilution by extracting $B_{\text{WA}}^{(0,1,2)}$ from a combined fit to the moments and the total rates (recall that $\bar{B}_{\text{WA}}^{(1,2)}$ are linear combinations of $B_{\text{WA}}^{(0,1,2)}$). The results for D^0 and D_s decays are shown in figure 3 and are consistent with $B_{\text{WA}}^{(0)} \approx B_{\text{WA}}^{(1)} \approx B_{\text{WA}}^{(2)}$, but the errors are too large to draw a conclusion concerning WA dilution.

Finally we study the correlation between ρ_D^3 and WA by considering ρ_D^3 a free parameter in the fit. In this way, the dominant source of uncertainty in the moments is removed. From the variance, first and second moments of D^0 we find the following constraints (the results from D^+ and D_s data are consistent)

$$\bar{B}_{\text{WA}}^{(\sigma),s} + 0.017\rho_D^3 = 0.0009(2)\text{GeV}^3, \quad \bar{B}_{\text{WA}}^{(2),s} + 0.021\rho_D^3 = 0.0010(5)\text{GeV}^3, \quad \bar{B}_{\text{WA}}^{(1),s} + 0.014\rho_D^3 = 0.0006(4)\text{GeV}^3.$$

Assuming no WA dilution one then obtains in each of these cases

$$B_{\text{WA}}^s + 0.043\rho_D^3 = 0.0021(5)\text{GeV}^3, \quad B_{\text{WA}}^s + 0.035\rho_D^3 = 0.0016(9)\text{GeV}^3, \quad B_{\text{WA}}^s + 0.033\rho_D^3 = 0.0014(10)\text{GeV}^3,$$

and it is remarkable that they all agree quite well. This may be viewed as a mild indication that there is no significant WA dilution, or possibly that the dilution is similar in the three cases. Indeed, assuming vanishing WA at $\mu_{\text{WA}} = 0.8\text{GeV}$ all the moments can be reproduced by $\rho_D^3(0.5\text{GeV}) = 0.05(1)\text{GeV}^3$.

In order to connect to $B \rightarrow X_u \ell \nu$ we refer to the WA matrix elements determined in charm decays as B_{WA}^{cq} and consider their relation to those relevant in B semileptonic decays, B_{WA}^{bq} , taking into account eq. (9). In the heavy

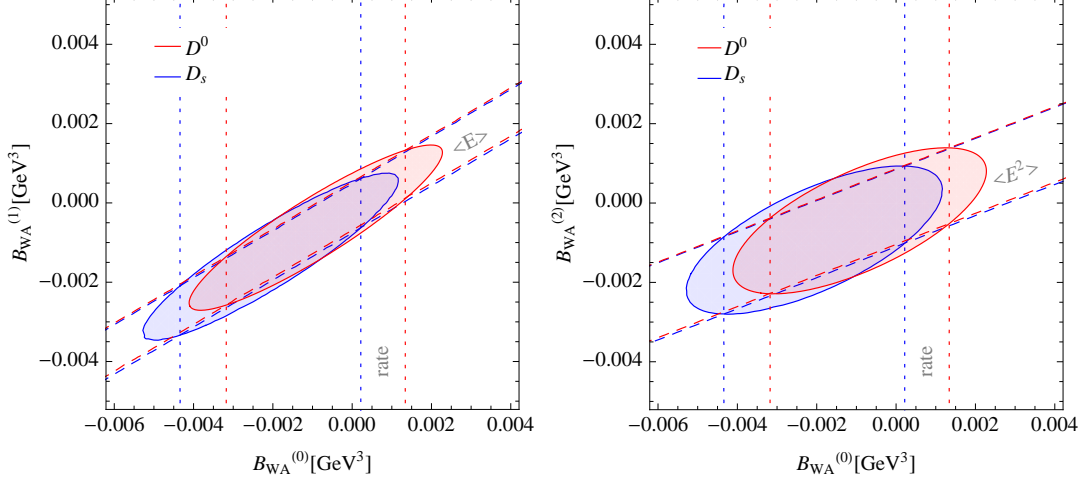


Figure 3: Combined constraints on $B_{WA}^{(i),s}$ from the total semileptonic rates of D^0 and D_s and the first two leptonic energy moments.

quark limit, $f_P \sim m_P^{-1/2}$ [35] so that $B_{P,i}$ scale as constants with heavy quark mass, but recent lattice results give $f_D \approx 0.21$ GeV, $f_B \approx 0.20$ GeV [36]. We also neglect any evolution of the WA operators and write

$$B_{WA}^{bq}(\mu_{WA}) = \frac{m_B f_B^2}{m_D f_D^2} B_{WA}^{cq}(\mu_{WA}). \quad (24)$$

The parametric enhancement due to meson masses and decay constants is a significant factor of 2.5. Due to finite heavy quark masses, one might also expect additional power corrections, which spoil the exact scaling of B_{WA} between the D and B sectors. From eqs. (23,24) we derive a bound

$$|B_{WA}^b(\mu_{WA} = 0.8\text{GeV})| \lesssim 0.006\text{GeV}^3, \quad (25)$$

which holds for the non-valence contributions, although we stress that the data seem to prefer even smaller values. The valence contribution is more constrained from the ratio of the D_s and D^0 rates and leptonic moments, from which we expect

$$-0.004\text{GeV}^3 \lesssim \Delta B_{WA}^b(\mu_{WA} = 0.8\text{GeV}) \lesssim 0.002\text{GeV}^3. \quad (26)$$

The bounds lead to a maximum 2% WA correction to the total rate of $B \rightarrow X_u \ell \nu$. In turn, this translates into an uncertainty of 1% on $|V_{ub}|$ extracted from the total rate and from the most inclusive experimental analyses, like those that involve a lower cut on the invariant hadronic mass. Our bound on the WA expectation value improves on previous estimates. In [27], for instance, the maximum value allowed for $B_{WA}^b(\mu_{WA} = 1\text{GeV})$ was as high as 0.020GeV^3 , or $B_{WA}^b(\mu_{WA} = 0.8\text{GeV}) = 0.018\text{GeV}^3$.

Our analysis is compatible with the one of ref. [14] for the valence contribution, although we allow for a larger SU(3) violation. The analysis of ref. [15] gives a determination of both valence and non-valence WA from the D semileptonic widths only. In our notation the results of ref. [15] correspond to a valence contribution $\Delta B_{WA}^{cs} = -0.0015(9)\text{GeV}^3$ and to an isosinglet contribution $B_{WA}^{cs} = 0.0036(5)\text{GeV}^3$. The valence contribution is perfectly compatible with our estimate from the widths only. The non-valence contribution, on the other hand, has a tiny error and should be compared with $B_{WA}^{cs} = -0.001(3)\text{GeV}^3$ that we extract from the widths only, see eq. (19). The difference is presumably due to various sources: *i*) the charm mass that we employ is higher than that employed in [15]: we start with the result of the global fit [29], which is in excellent agreement with other m_c determinations like charmonium sum rules, and evolve it to lower μ_{kin} ; *ii*) our OPE parameters (taken from [29] and then evolved to lower μ_{kin}) are different from those of ref. [15], which may lead to sizable differences in charm but not in bottom decays; *iii*) the method of [15] implies the use of $\alpha_s(m_b)$, and might underestimate the perturbative corrections. The discrepancy between the two determinations provides indeed additional motivation for using the moments to constrain WA, as they are less sensitive to power and perturbative corrections. In any case, we find remarkable that our determination from the moments is compatible with that from the widths and relatively stable wrt changes in the definition of the charm

mass or in μ_{kin} . Notice also that despite the larger value of B_{WA}^{cs} they find, the authors of [15] end up with an estimate for the WA effects in the $B \rightarrow X_u \ell \nu$ rate which is very similar to ours, once the enhancement related to eq. (24) is taken into account. This is because our large uncertainty almost covers the discrepancy in the central values.

V. SUMMARY

We have performed an analysis of inclusive semileptonic D decays using the Heavy Quark Expansion. In addition to the total widths, we have used the Cleo data on the lepton energy spectra to compute the first few moments. The latter are quite sensitive probes of possible Weak Annihilation contributions, both in its isosinglet and isotriplet components, and determine very precisely a linear combination of the expectation values of the Darwin and WA operators. We have shown that the extraction of WA from the moments depends to the way WA is distributed in the lepton energy spectrum and we have taken this effect into account in our error estimates.

Our analysis of Cleo data shows no evidence for Weak Annihilation, i.e. the OPE describes well the experimental results even in the absence of WA. There is a mild indication of non-zero valence WA in the ratio of the D_s and D^0 widths, but it does not seem to be supported by the ratio of the lepton moments in D_s and D^0 decays. A possible explanation involves sizable $SU(3)$ breaking in the matrix elements of the power suppressed operators and/or a WA contribution which is broadly distributed over the leptonic spectrum.

We derive an upper limit on both valence and non-valence WA components, which allows us to put a bound of 2% on their relevance in the $B \rightarrow X_u \ell \nu$ decay rate and even less for the isotriplet component. We look forward to the individual measurements of the B^+ and B^0 charmless inclusive semileptonic decays, which could test our conclusions regarding the suppression of valence WA in these decays.

Finally we note that the hadronic mass and q^2 moments in inclusive semileptonic decays of heavy quarks are quite sensitive to non-perturbative contributions, see e.g. [26]. Their measurement at Cleo-c or at the recently started BESIII experiment [37] might help us disentangle the various non-perturbative effects.

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